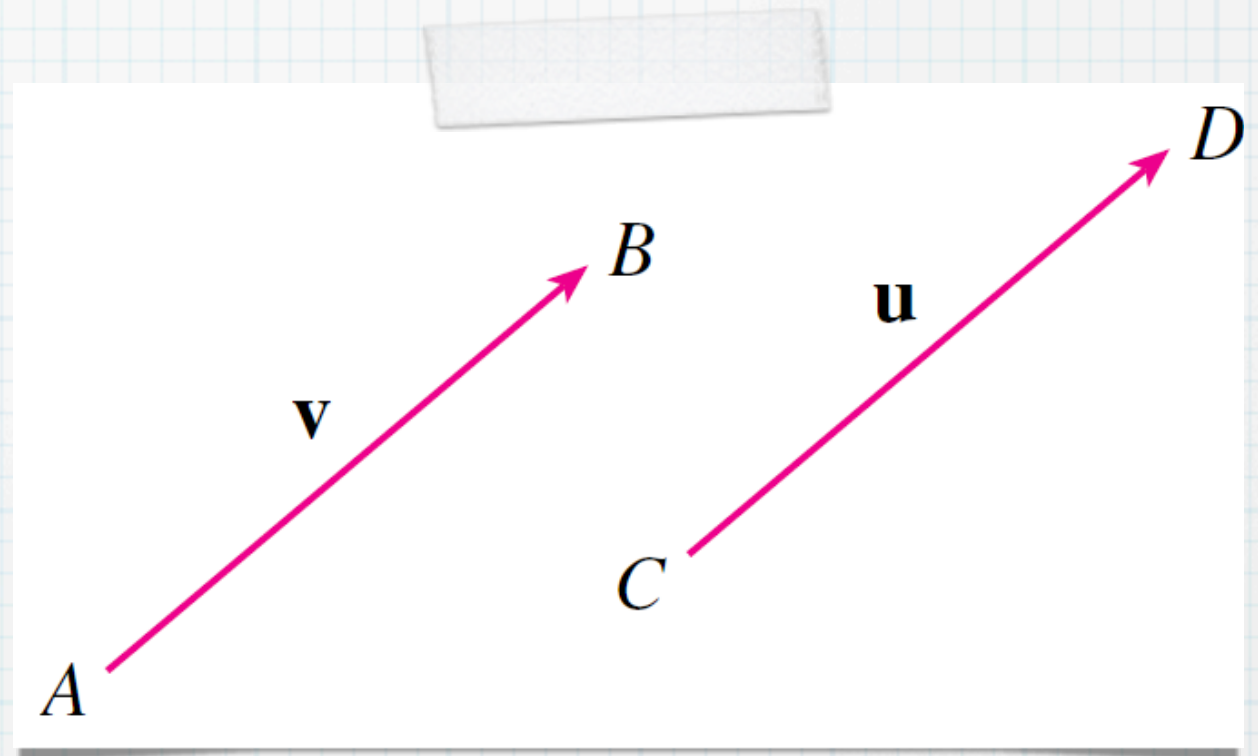


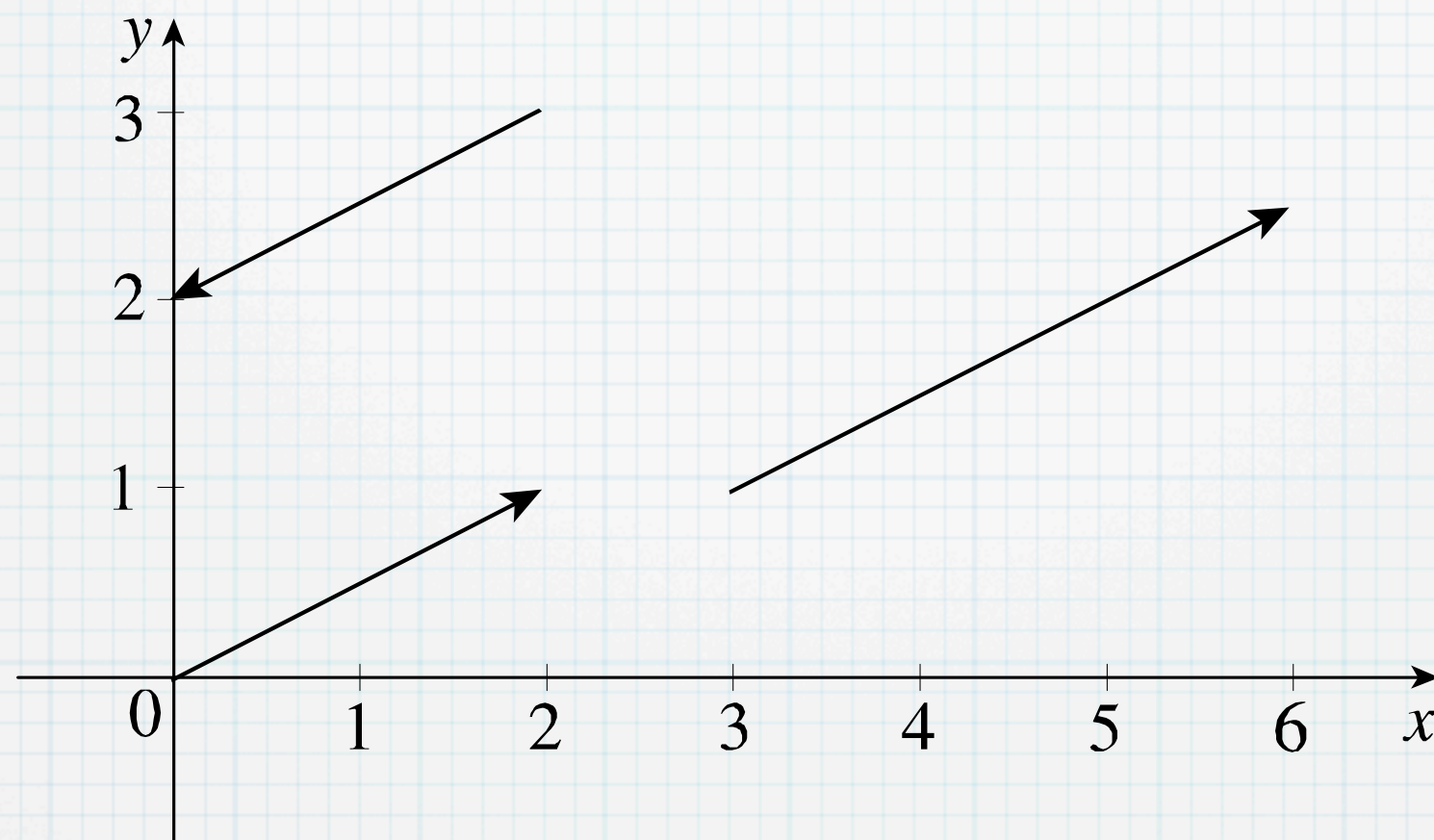
Vectors

Section 9.2

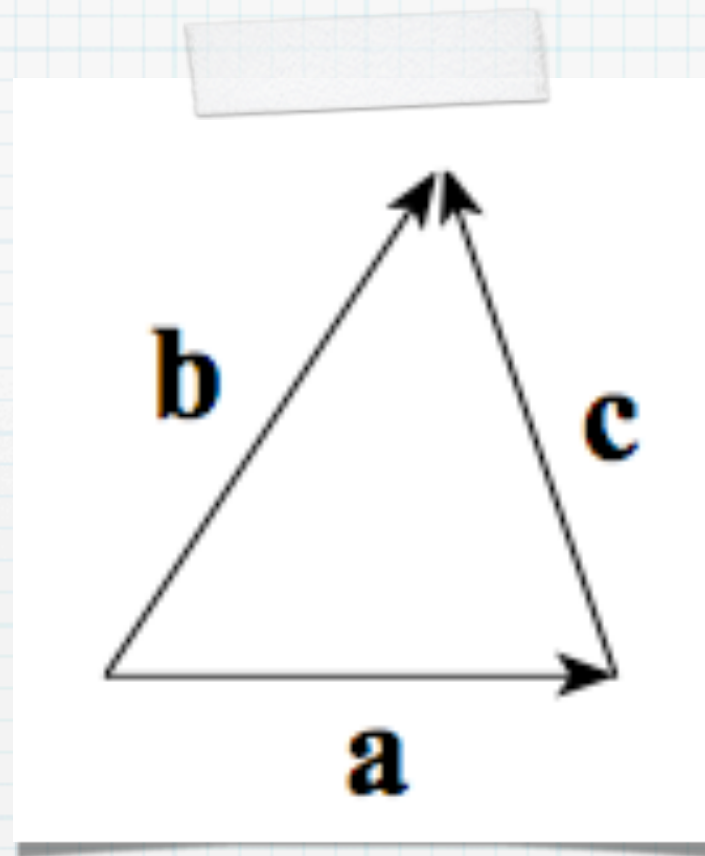
Vectors

- * A vector is a quantity (such as displacement or velocity or force) that has both magnitude and direction.
- * A vector is often represented by an arrow or a directed line segment.
- * The length of the arrow represents the magnitude of the vector and
- * the arrow points in the direction of the vector.

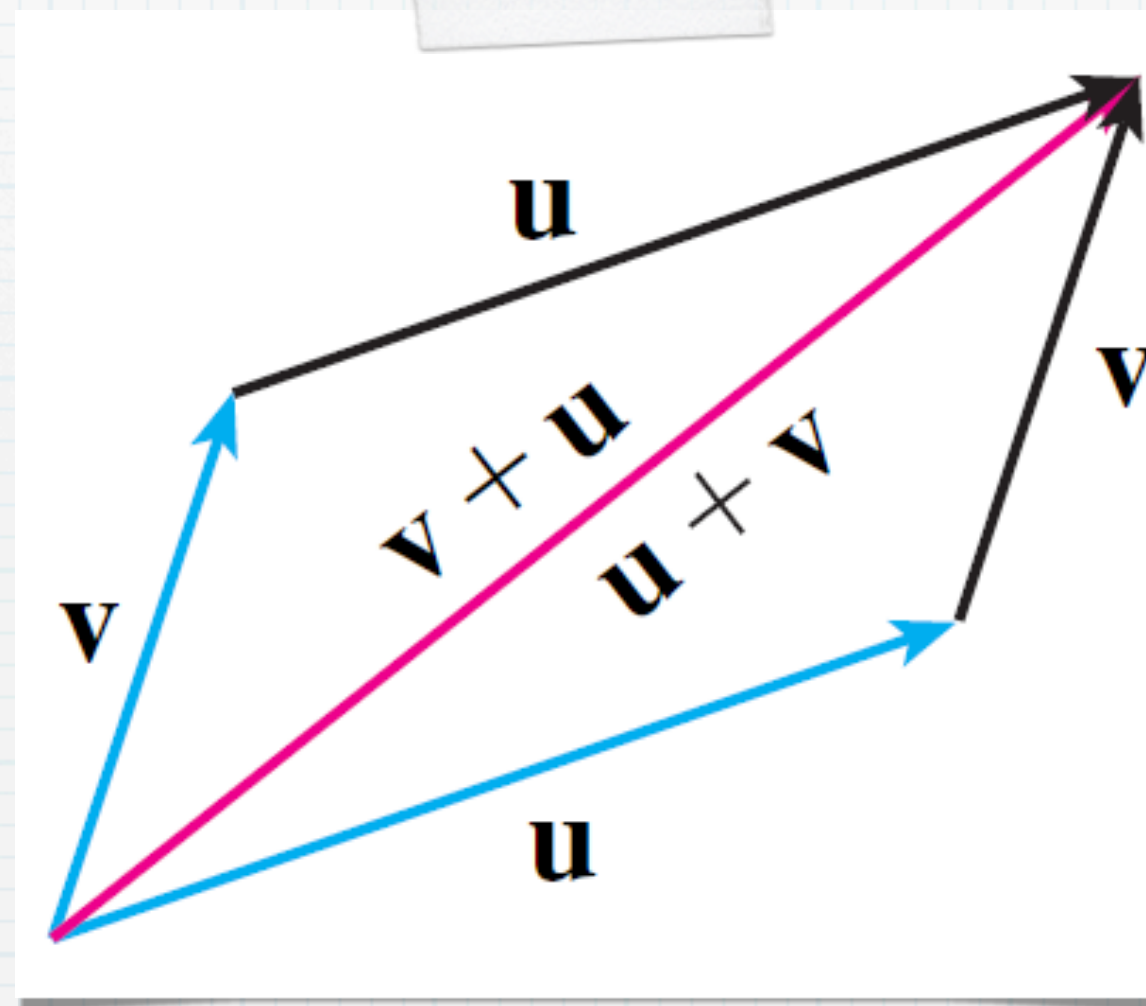




Parallel?



Describe c in terms of a and b

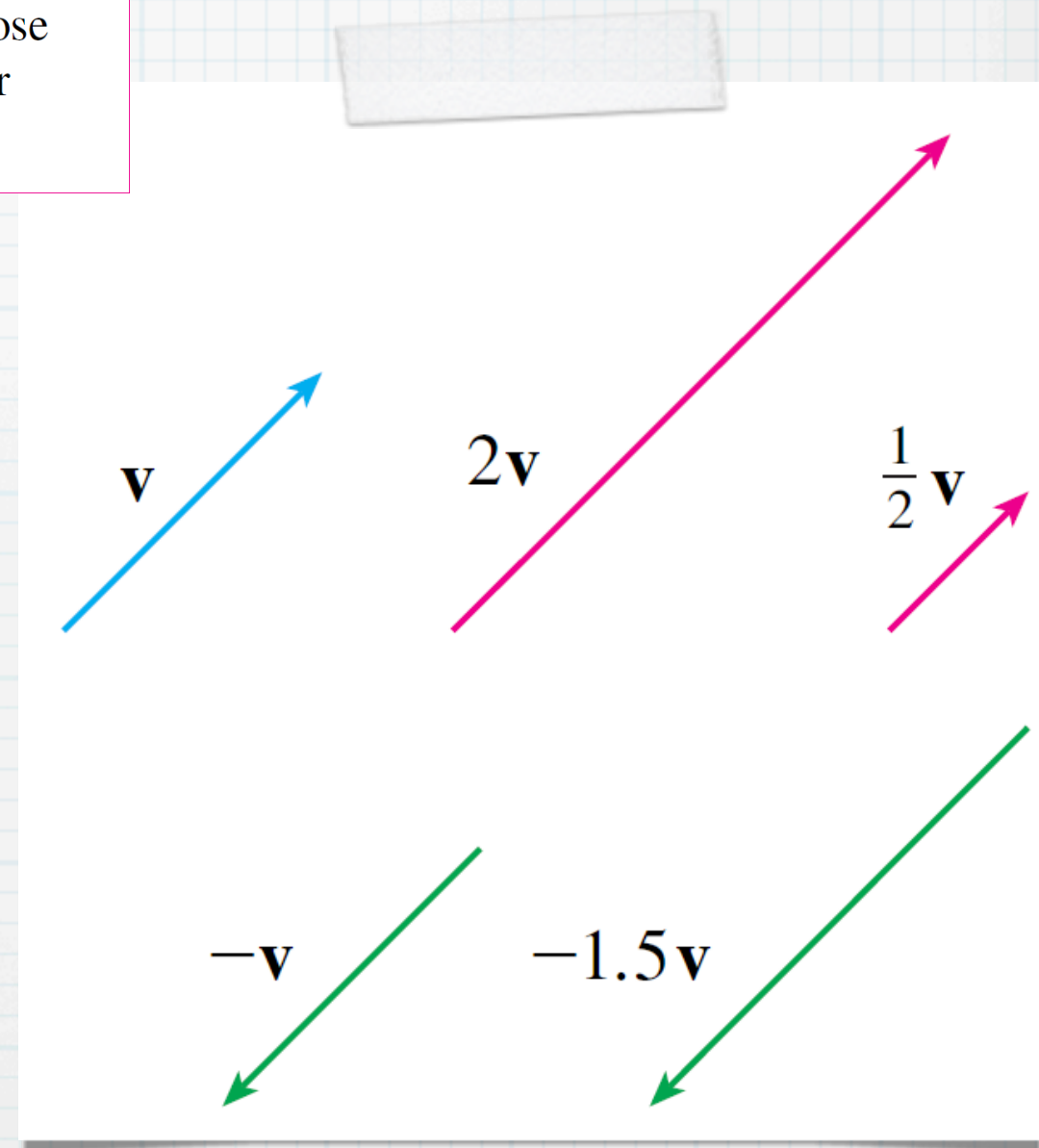


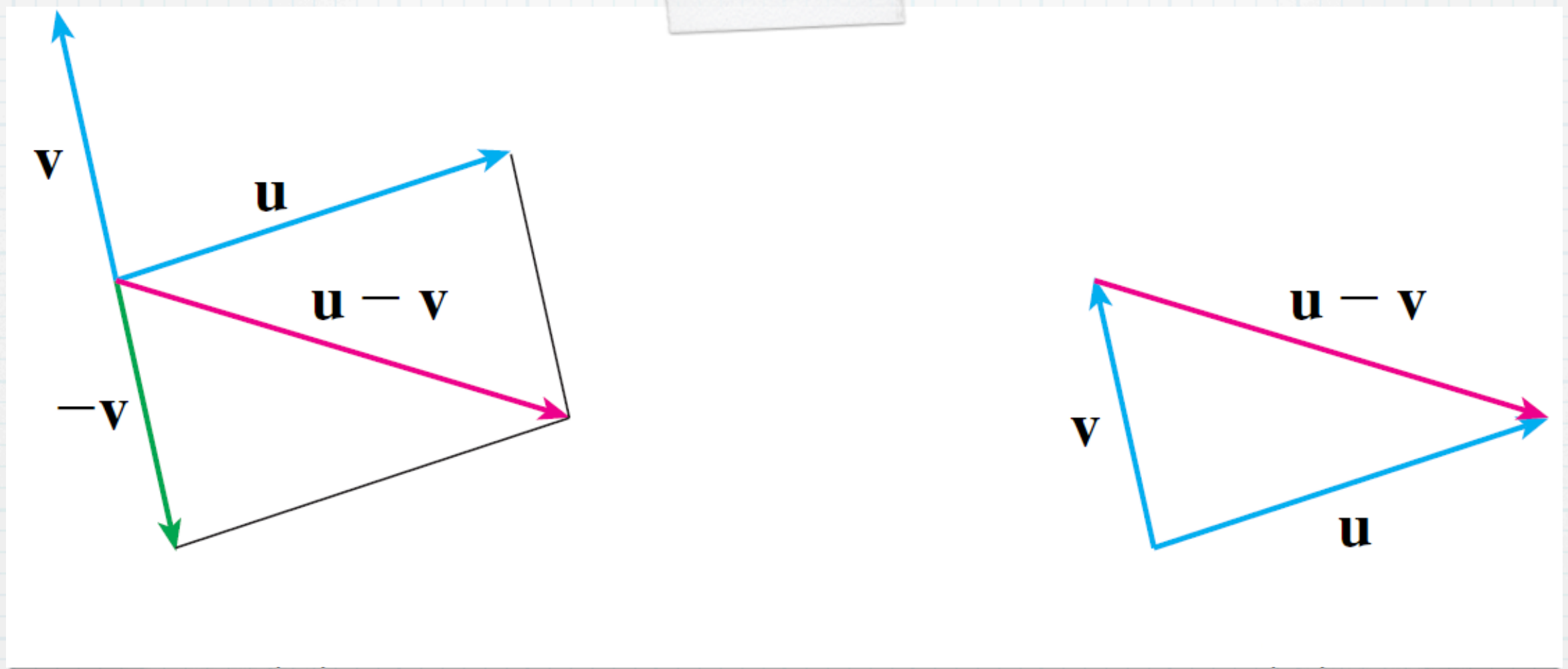
Vector Addition

Definition of Vector Addition If \mathbf{u} and \mathbf{v} are vectors positioned so the initial point of \mathbf{v} is at the terminal point of \mathbf{u} , then the **sum** $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of \mathbf{u} to the terminal point of \mathbf{v} .

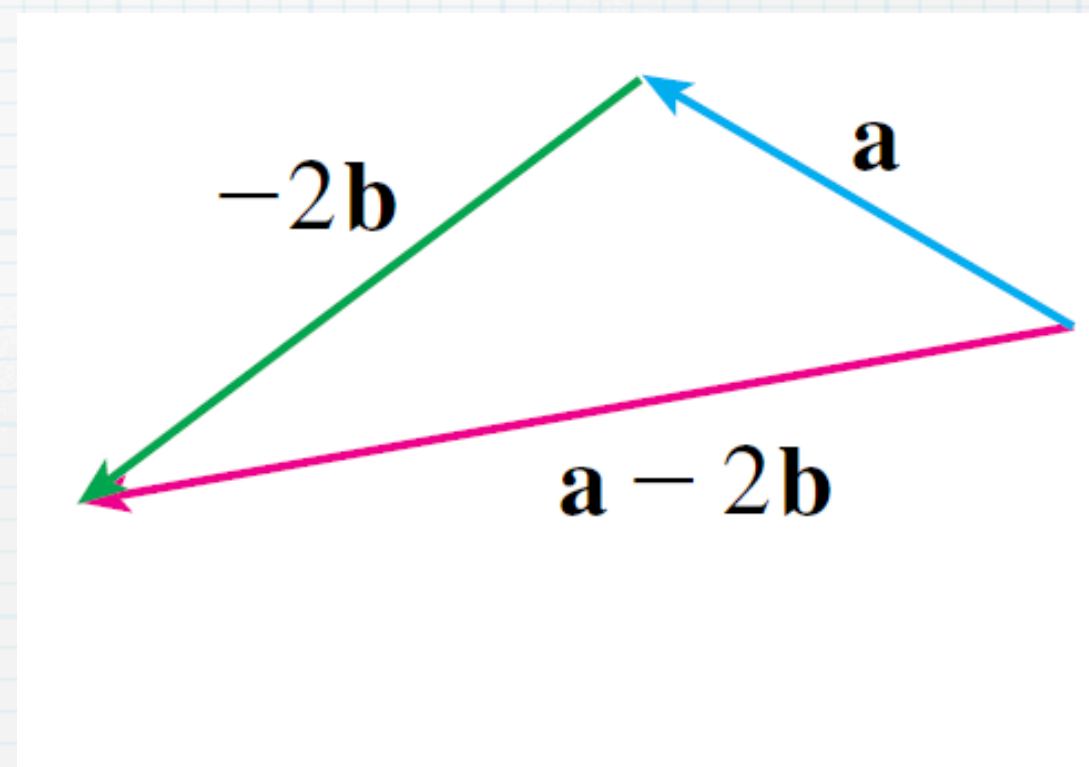
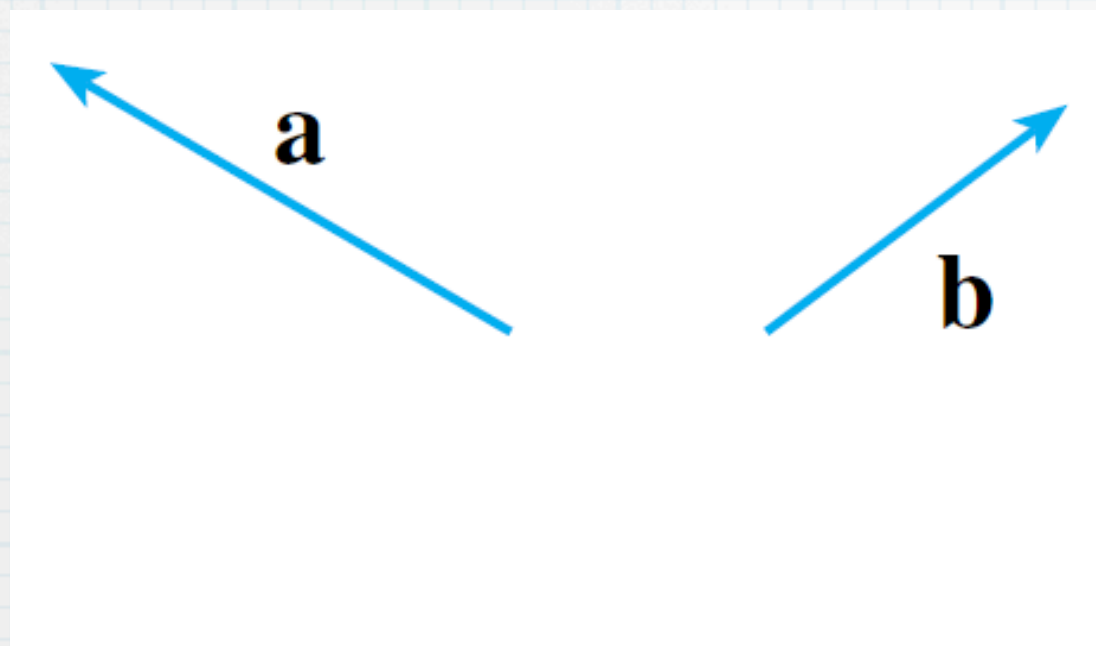
Scalar Multiplication

Definition of Scalar Multiplication If c is a scalar and \mathbf{v} is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of \mathbf{v} and whose direction is the same as \mathbf{v} if $c > 0$ and is opposite to \mathbf{v} if $c < 0$. If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = \mathbf{0}$.



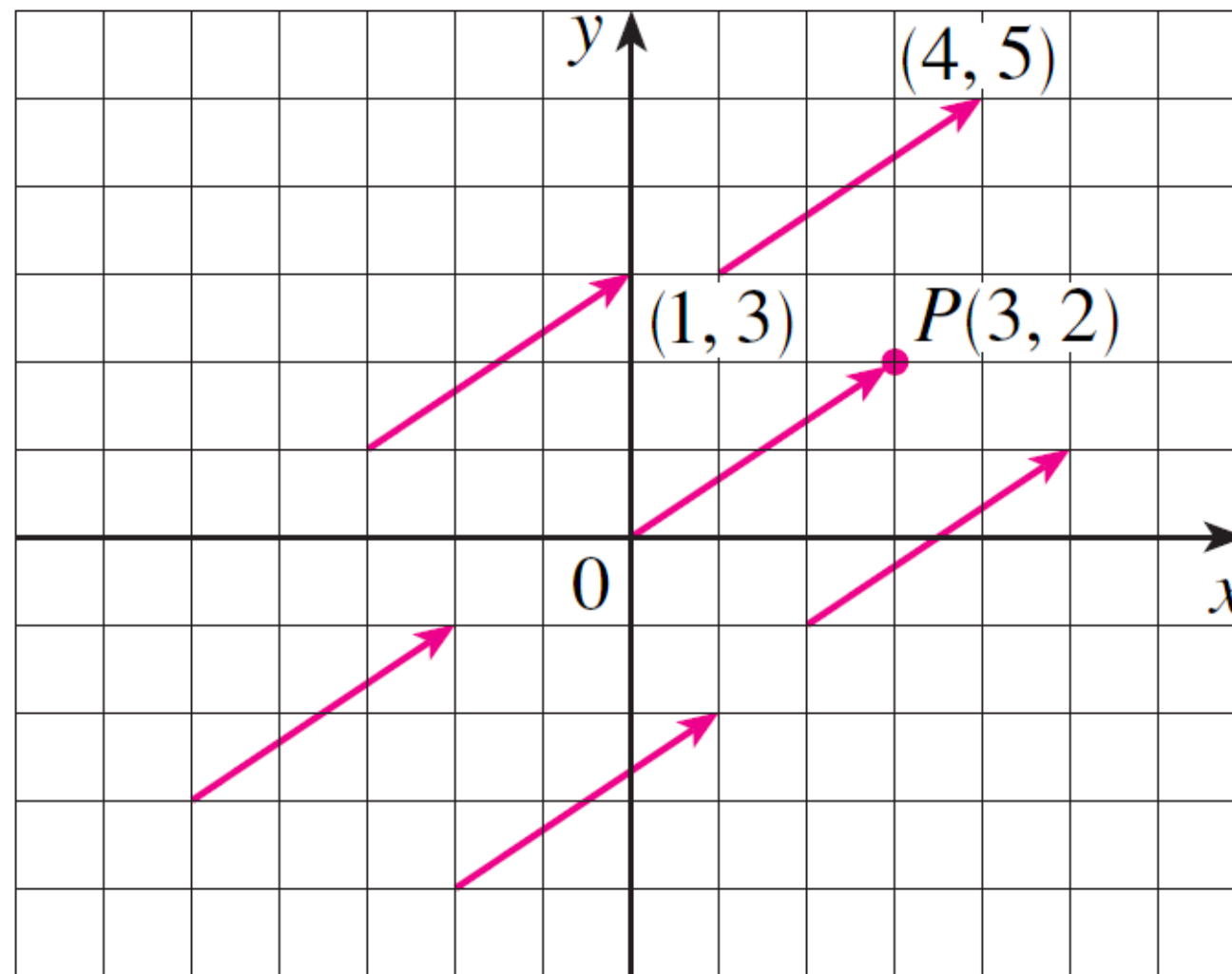


Vector Subtraction



Components

$$\mathbf{a} = \langle a_1, a_2 \rangle \quad \text{or} \quad \mathbf{a} = \langle a_1, a_2, a_3 \rangle$$



Representations of the vector $\mathbf{a} = \langle 3, 2 \rangle$

Length

The length of the two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

The length of the three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Using Components

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \qquad \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

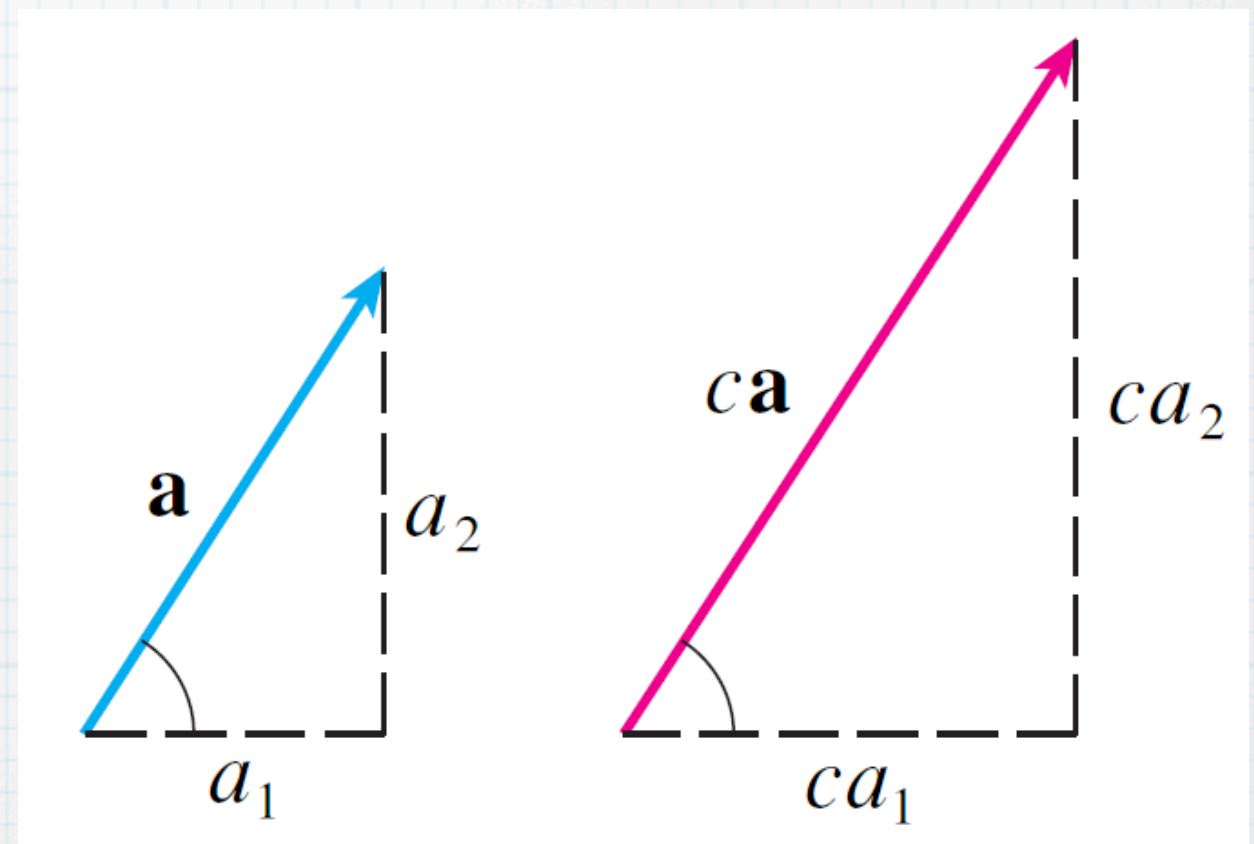
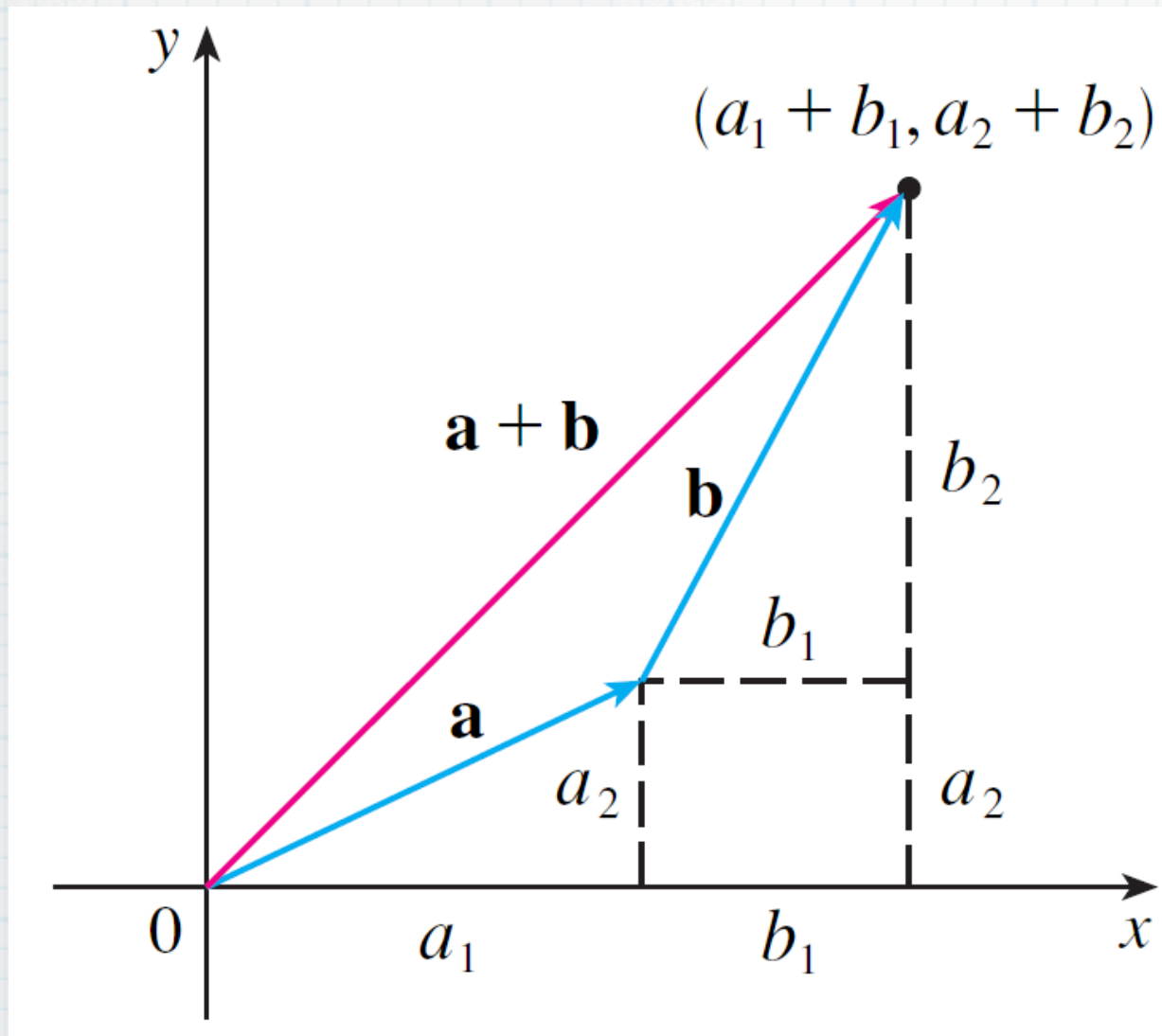
Similarly, for three-dimensional vectors,

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

Graphically



Properties of Vectors

Properties of Vectors If \mathbf{a} , \mathbf{b} , and \mathbf{c} are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$

3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$

4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$

5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$

6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$

7. $(cd)\mathbf{a} = c(d\mathbf{a})$

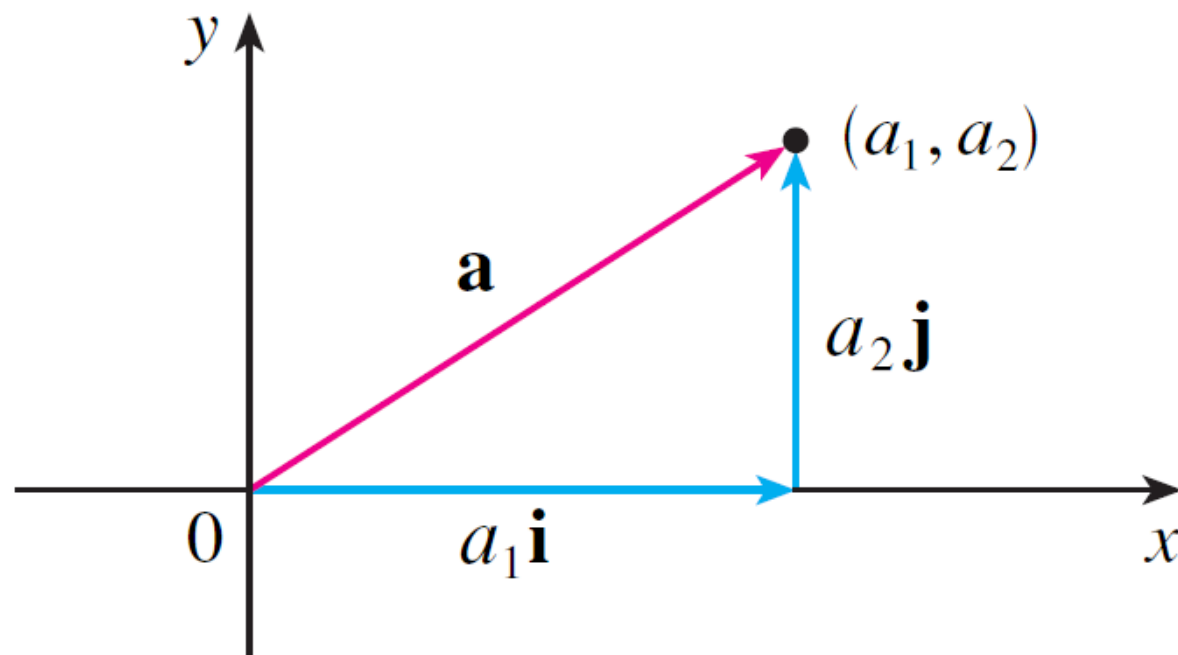
8. $1\mathbf{a} = \mathbf{a}$

Standard Bases

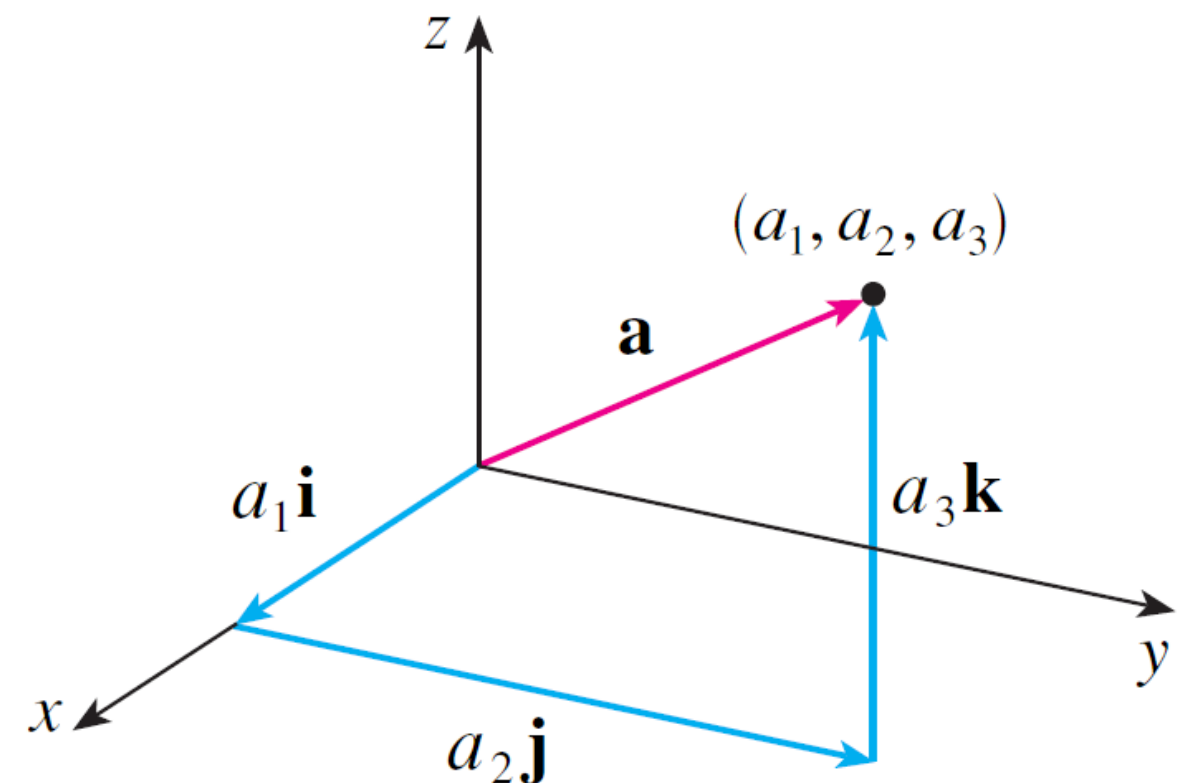
$$\mathbf{i} = \langle 1, 0, 0 \rangle$$

$$\mathbf{j} = \langle 0, 1, 0 \rangle$$

$$\mathbf{k} = \langle 0, 0, 1 \rangle$$



$$(a) \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$$



$$(b) \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

Handout

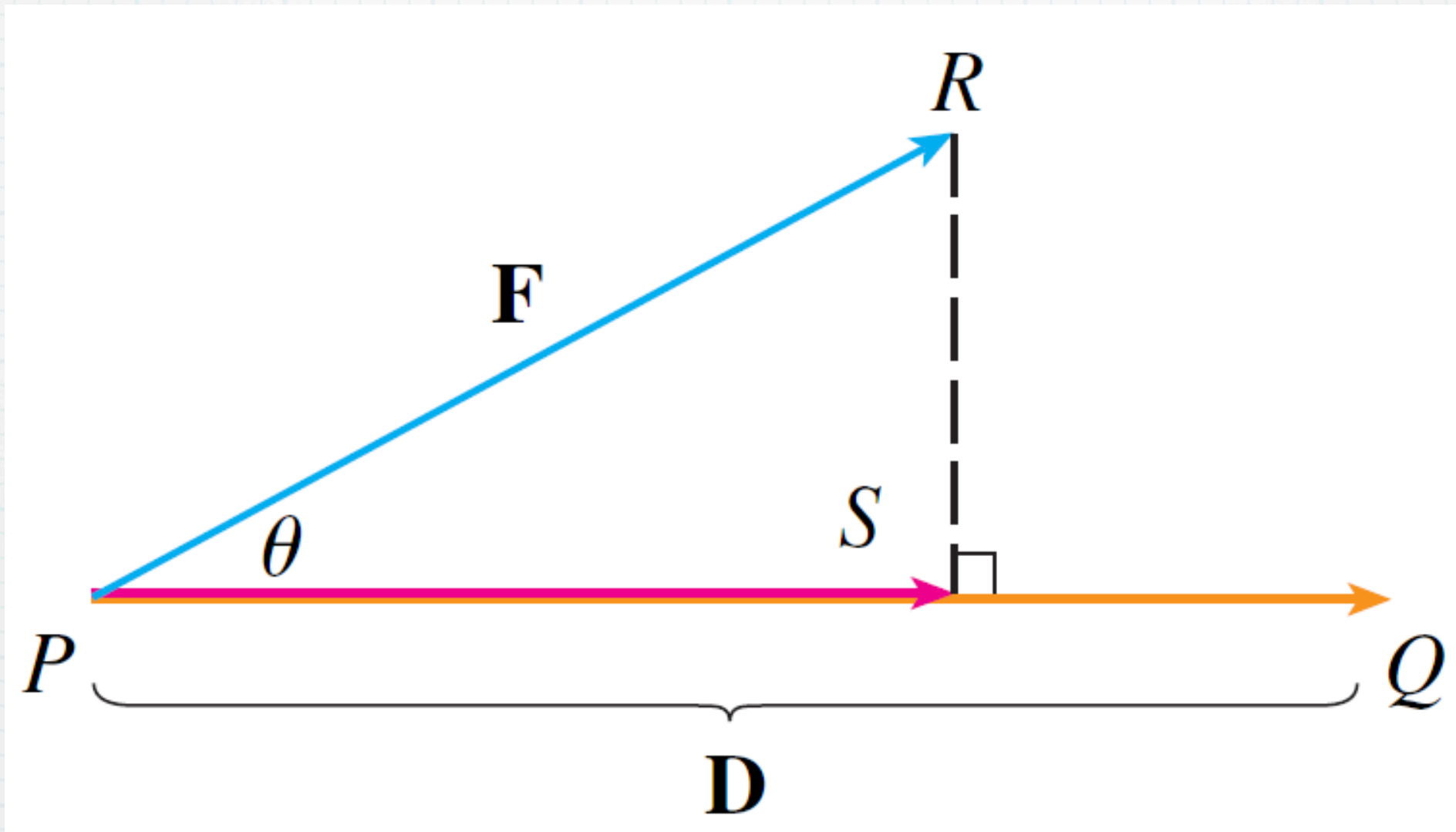
- * Component Vectors
- * Where do they point? Group Work 3

Dot Product

Section 9.3

Work

Work done by a constant force F (in the line of motion) in moving an object through a distance d : $W = Fd$



Need to know the component of the force F in direction of motion

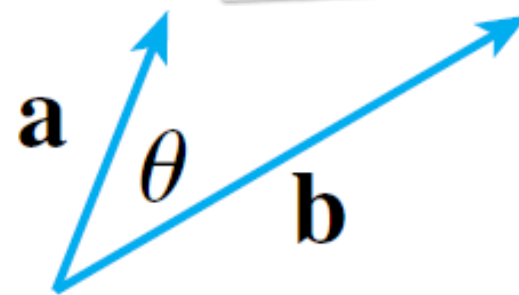
Dot Product

Definition The **dot product** of two nonzero vectors **a** and **b** is the number

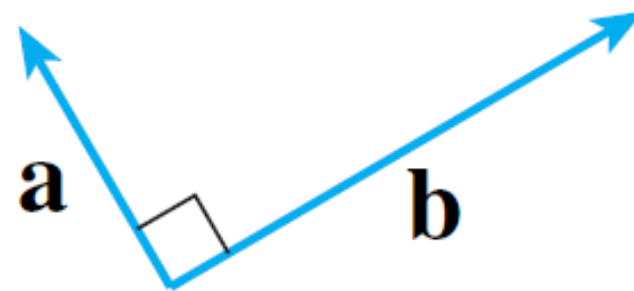
$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

where θ is the angle between **a** and **b**, $0 \leq \theta \leq \pi$. (So θ is the smaller angle between the vectors when they are drawn with the same initial point.) If either **a** or **b** is **0**, we define $\mathbf{a} \cdot \mathbf{b} = 0$.

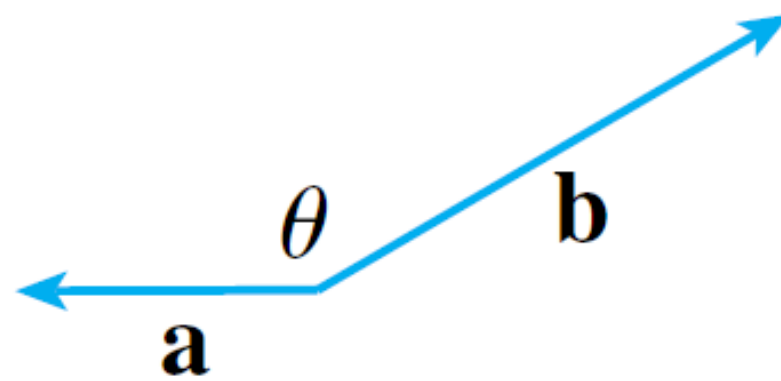
2 Two vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$.



$$\mathbf{a} \cdot \mathbf{b} > 0$$



$$\mathbf{a} \cdot \mathbf{b} = 0$$



$$\mathbf{a} \cdot \mathbf{b} < 0$$

Dot Product: Component Form

The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\langle 2, 4 \rangle \cdot \langle 3, -1 \rangle = 2(3) + 4(-1) = 2$$

$$\langle -1, 7, 4 \rangle \cdot \langle 6, 2, -\frac{1}{2} \rangle = (-1)(6) + 7(2) + 4(-\frac{1}{2}) = 6$$

$$(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \cdot (2\mathbf{j} - \mathbf{k}) = 1(0) + 2(2) + (-3)(-1) = 7$$

Properties

Properties of the Dot Product If \mathbf{a} , \mathbf{b} , and \mathbf{c} are vectors in V_3 and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$

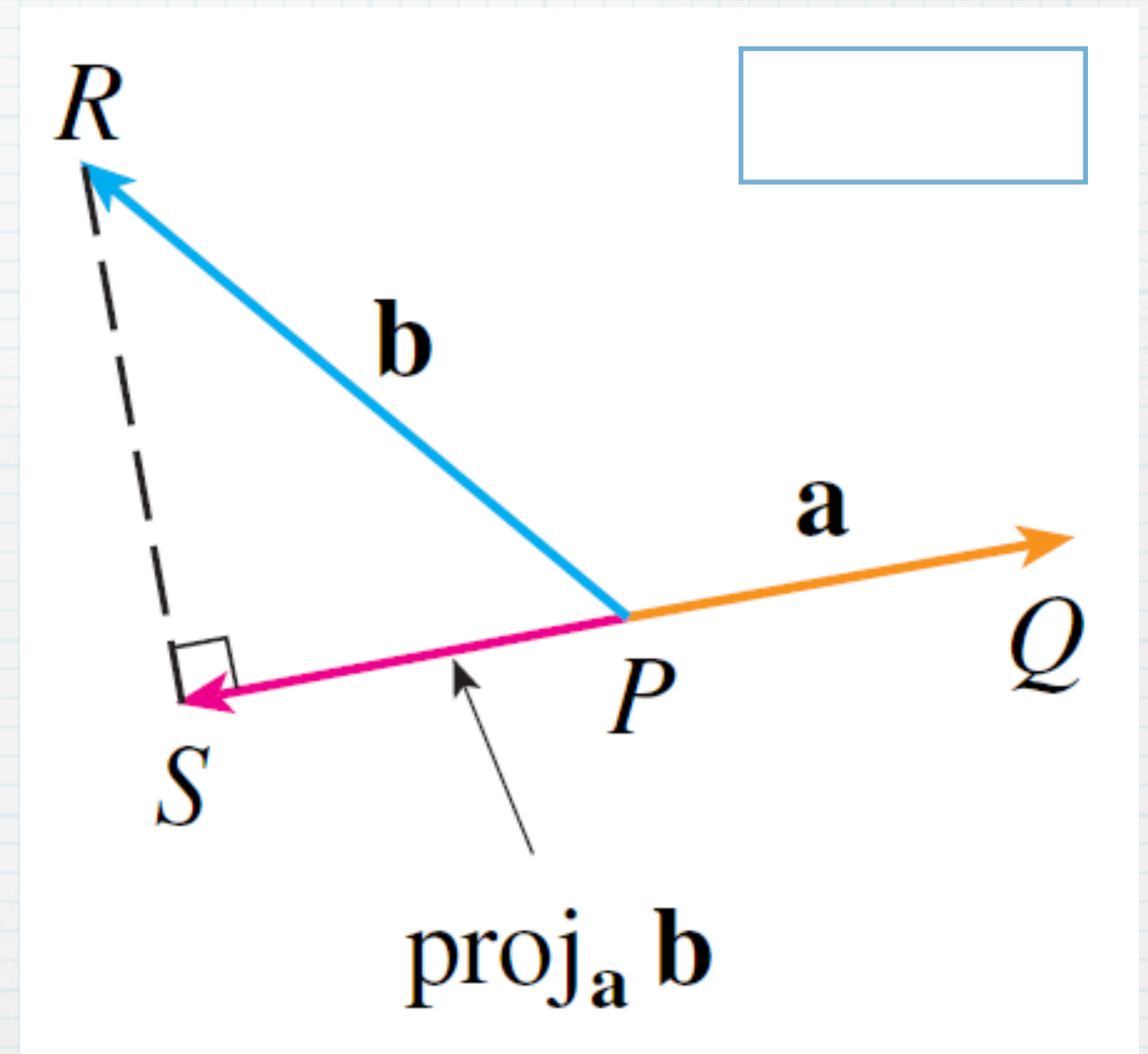
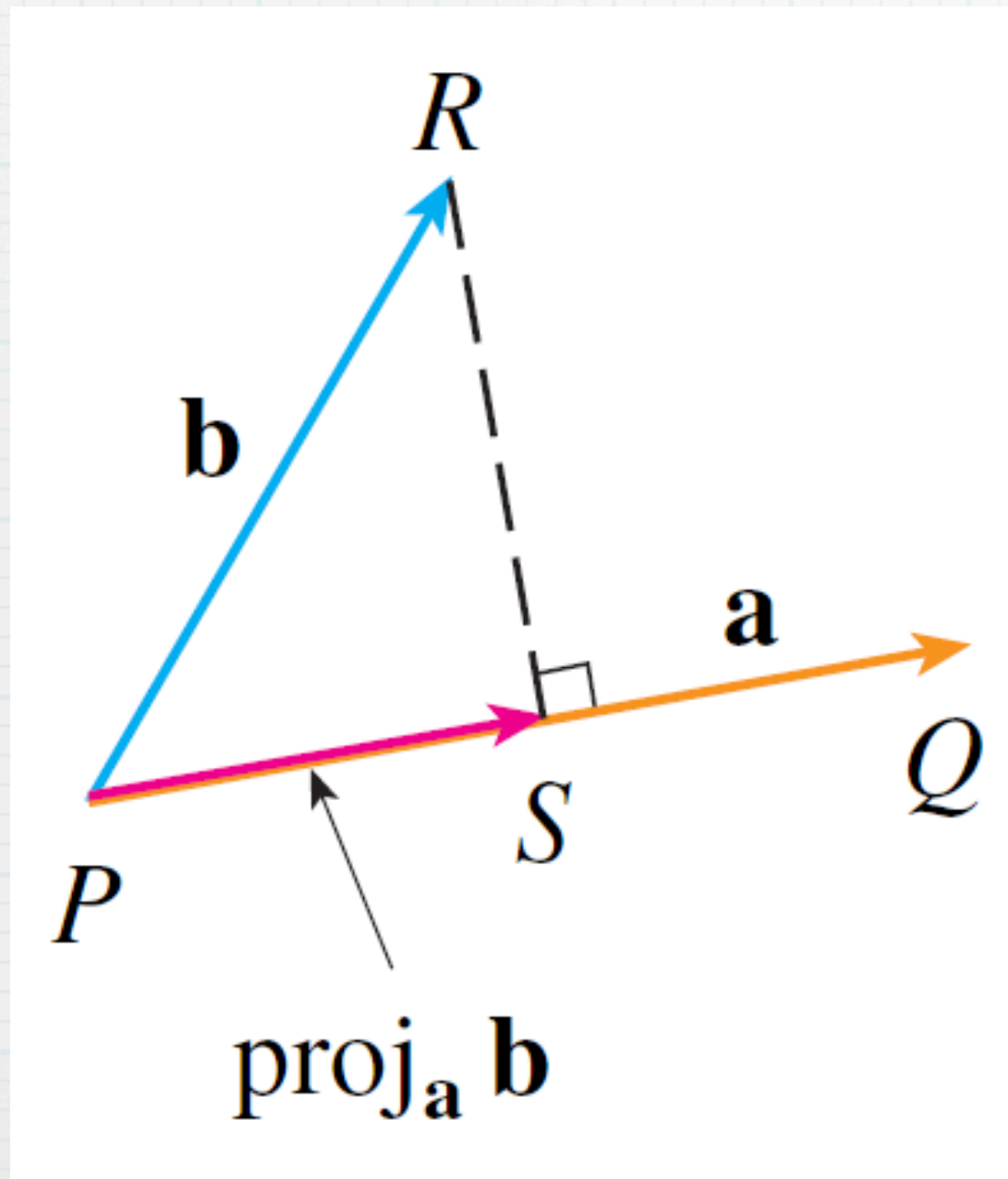
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

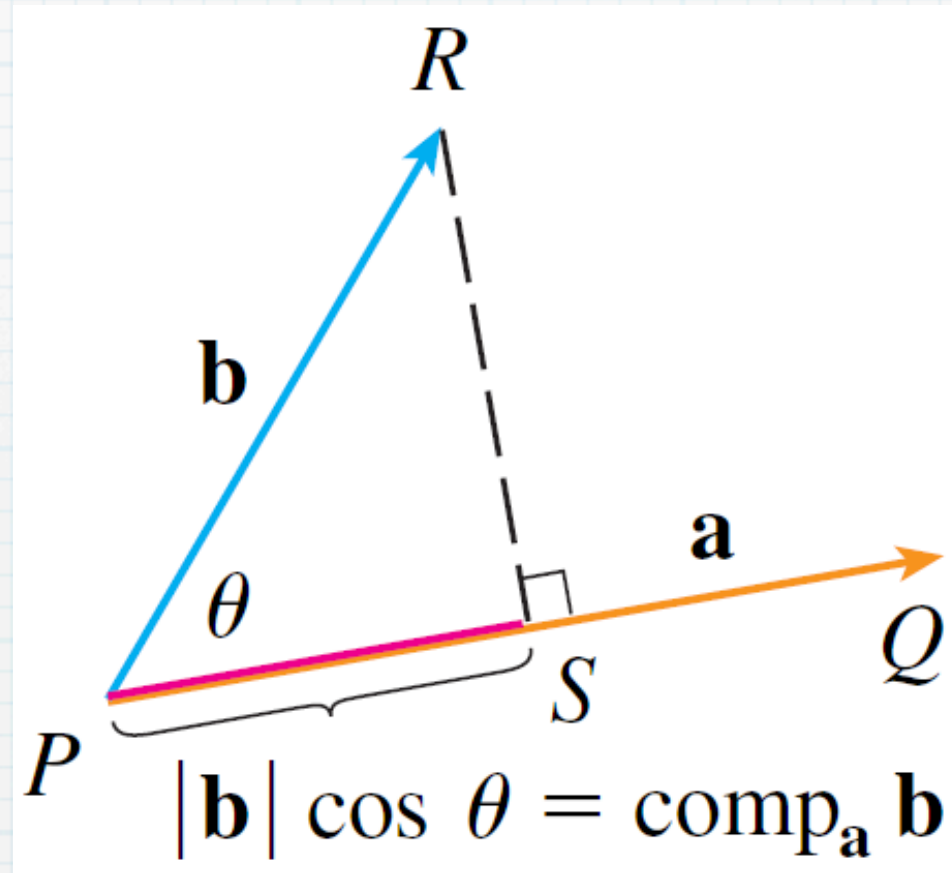
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$

5. $\mathbf{0} \cdot \mathbf{a} = 0$

Projections



Projections



Scalar projection of \mathbf{b} onto \mathbf{a} : $\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

Vector projection of \mathbf{b} onto \mathbf{a} : $\text{proj}_{\mathbf{a}} \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \right) \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$

Handouts

- * Regular Hexagons
- * The Right Stuff

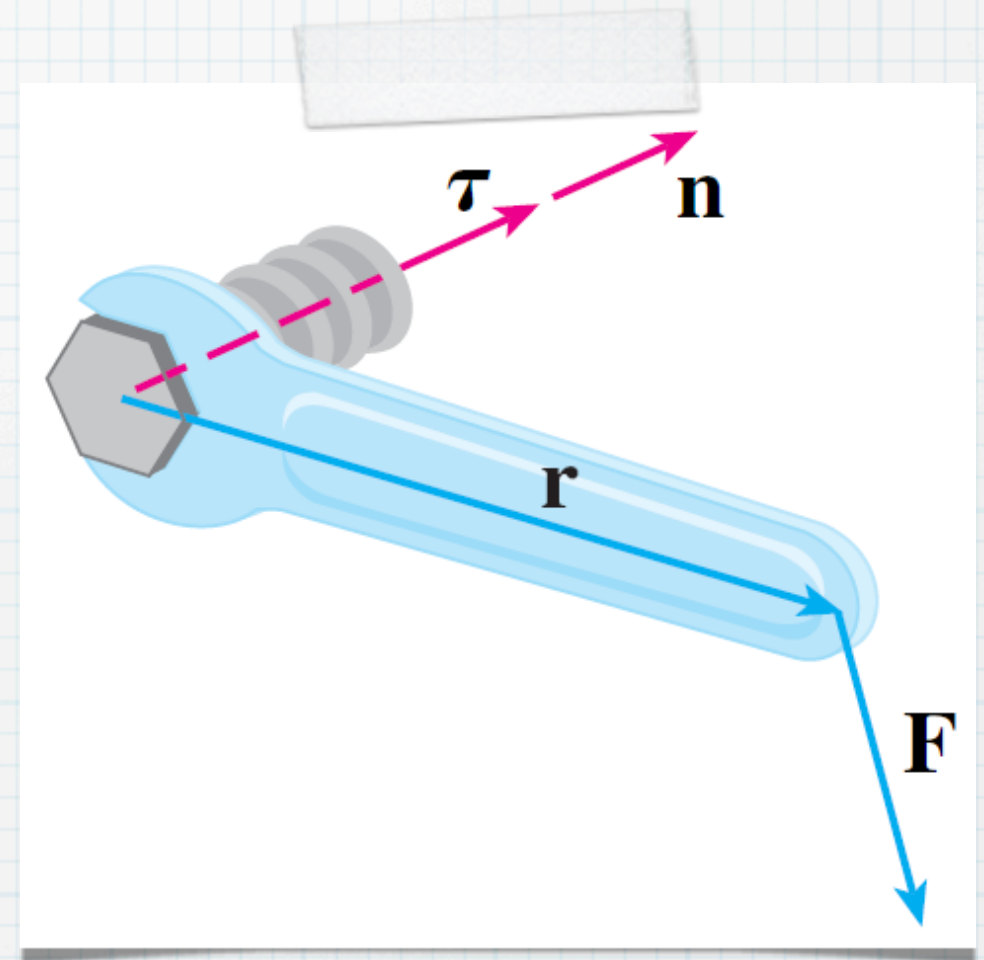
Cross Product

Section 9.4

Torque

Depends on

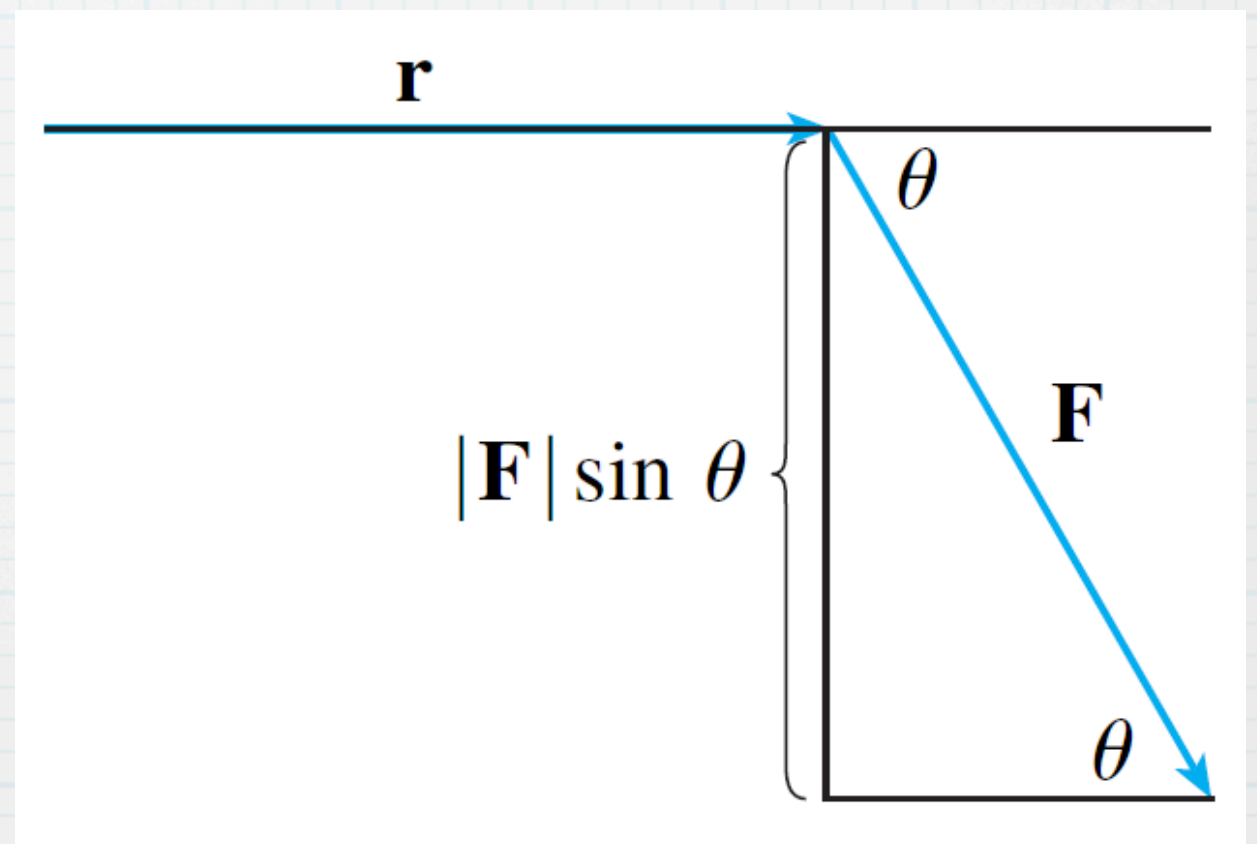
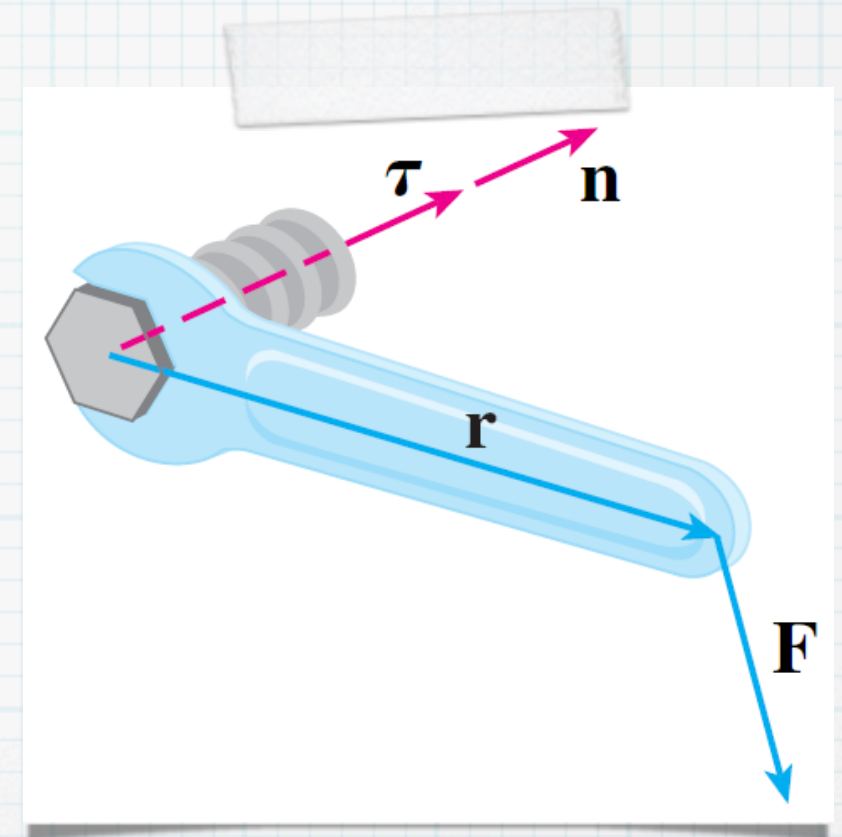
- * Distance from axis of bolt to point at which Force is applied:
 $|r|$ = magnitude of vector r
- * Scalar component of Force in the direction perpendicular to r .
This is the only component of the Force which can cause a rotation.



Torque

Depends on

- * Distance from axis of bolt to point at which Force is applied:
 $|r|$ = magnitude of vector r
- * Scalar component of Force in the direction perpendicular to r .
This is the only component of the Force which can cause a rotation.



$$|\tau| = |r| |F| \sin \theta$$

Cross Product

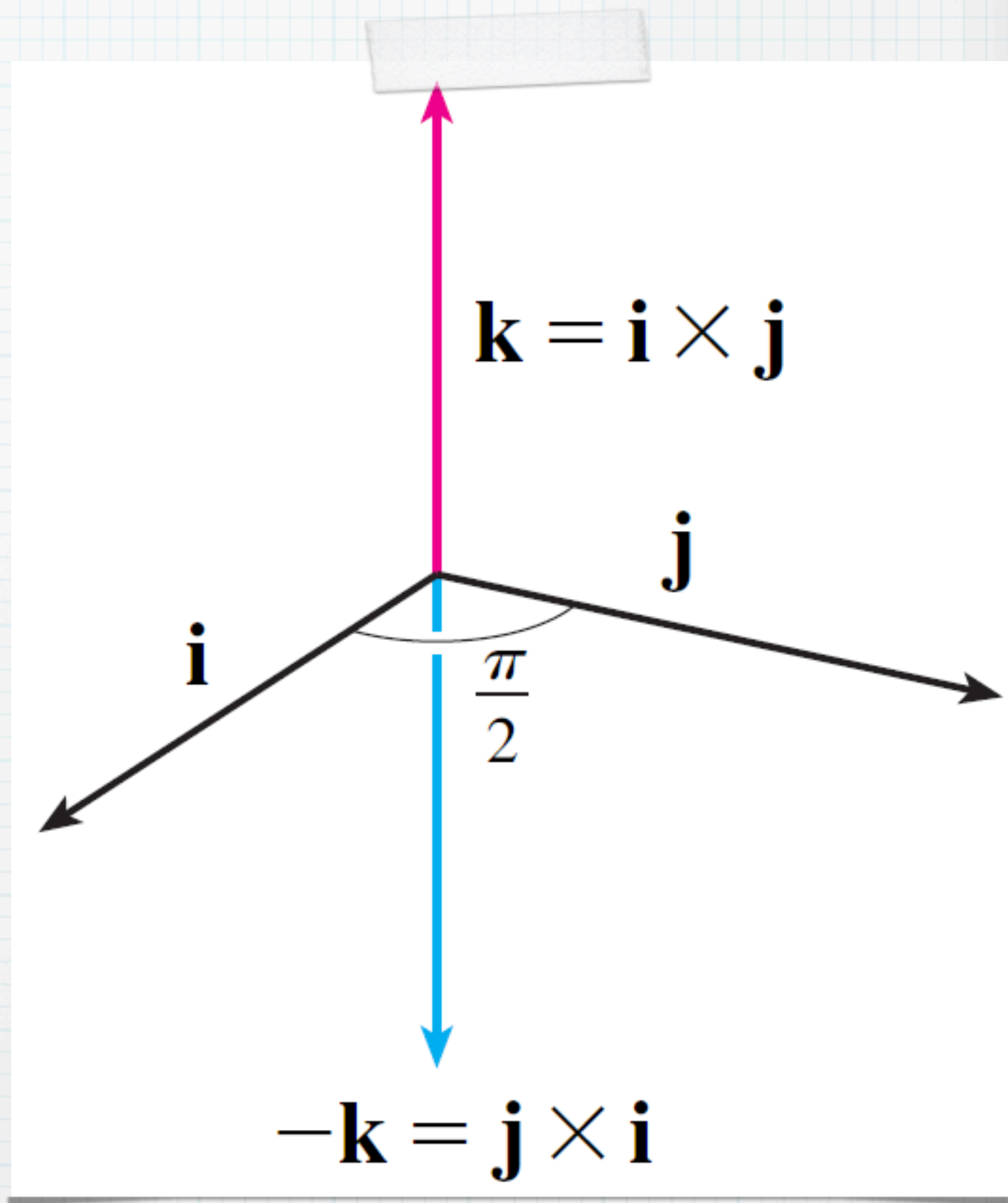
Definition If \mathbf{a} and \mathbf{b} are nonzero three-dimensional vectors, the **cross product** of \mathbf{a} and \mathbf{b} is the vector

$$\mathbf{a} \times \mathbf{b} = (|\mathbf{a}| |\mathbf{b}| \sin \theta) \mathbf{n}$$

where θ is the angle between \mathbf{a} and \mathbf{b} , $0 \leq \theta \leq \pi$, and \mathbf{n} is a unit vector perpendicular to both \mathbf{a} and \mathbf{b} and whose direction is given by the **right-hand rule**: If the fingers of your right hand curl through the angle θ from \mathbf{a} and \mathbf{b} , then your thumb points in the direction of \mathbf{n} . (See Figure 3.)

Cross Product:

Right Hand Rule

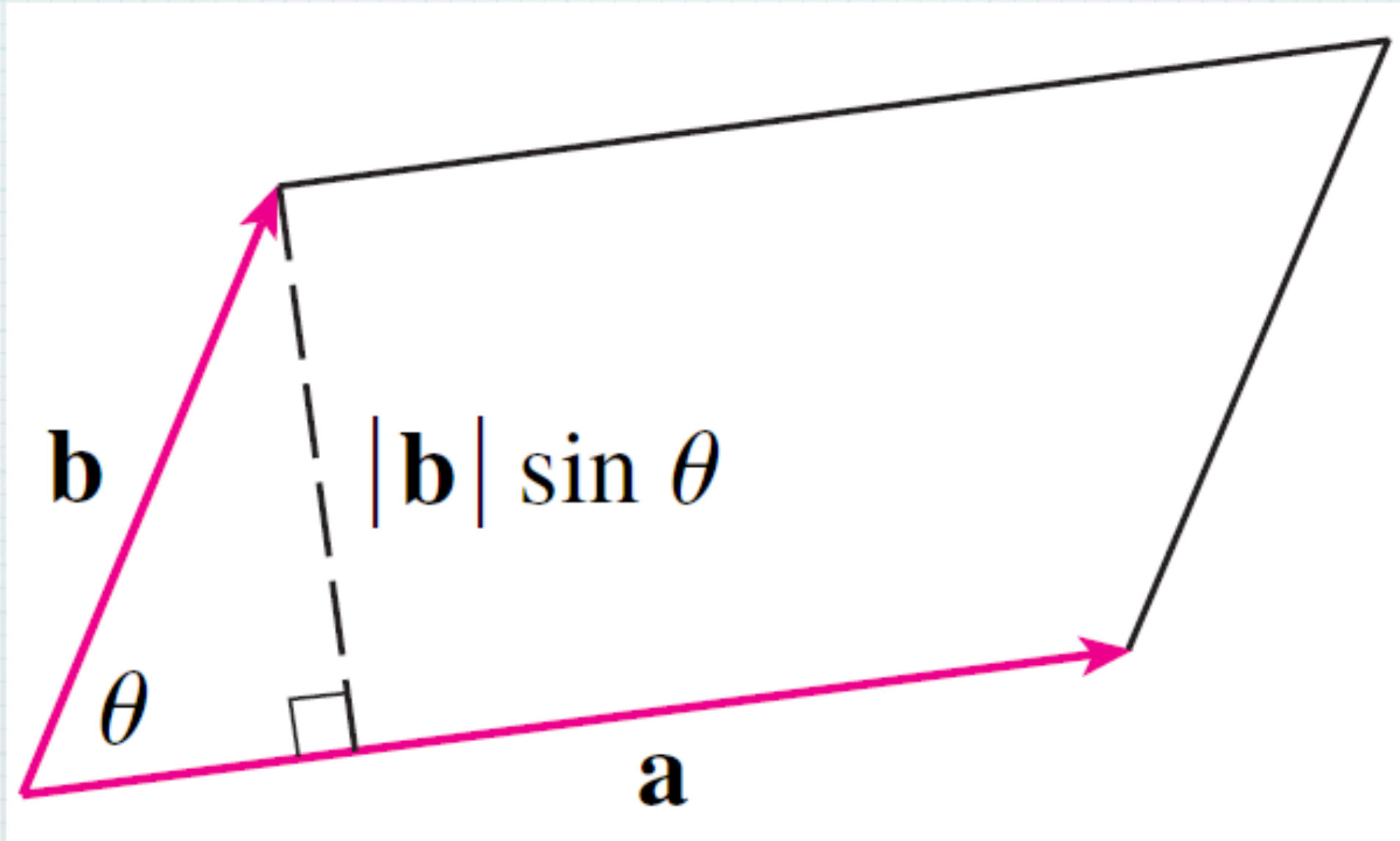


Cross Product: Properties

Properties of the Cross Product If **a**, **b**, and **c** are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$

Geometry



The length of the cross product $\mathbf{a} \times \mathbf{b}$ is equal to the area of the parallelogram determined by \mathbf{a} and \mathbf{b} .

Computing the Cross Product

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

$$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

$$\mathbf{a} \times \mathbf{b} = (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \times (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k})$$

$$= a_1b_1\mathbf{i} \times \mathbf{i} + a_1b_2\mathbf{i} \times \mathbf{j} + a_1b_3\mathbf{i} \times \mathbf{k}$$

$$+ a_2b_1\mathbf{j} \times \mathbf{i} + a_2b_2\mathbf{j} \times \mathbf{j} + a_2b_3\mathbf{j} \times \mathbf{k}$$

$$+ a_3b_1\mathbf{k} \times \mathbf{i} + a_3b_2\mathbf{k} \times \mathbf{j} + a_3b_3\mathbf{k} \times \mathbf{k}$$

$$= a_1b_2\mathbf{k} + a_1b_3(-\mathbf{j}) + a_2b_1(-\mathbf{k}) + a_2b_3\mathbf{i} + a_3b_1\mathbf{j} + a_3b_2(-\mathbf{i})$$

$$= (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

2 If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$$\mathbf{a} \times \mathbf{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle$$

Computing the Cross Product: Determinant Form

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \mathbf{k}$$

Example

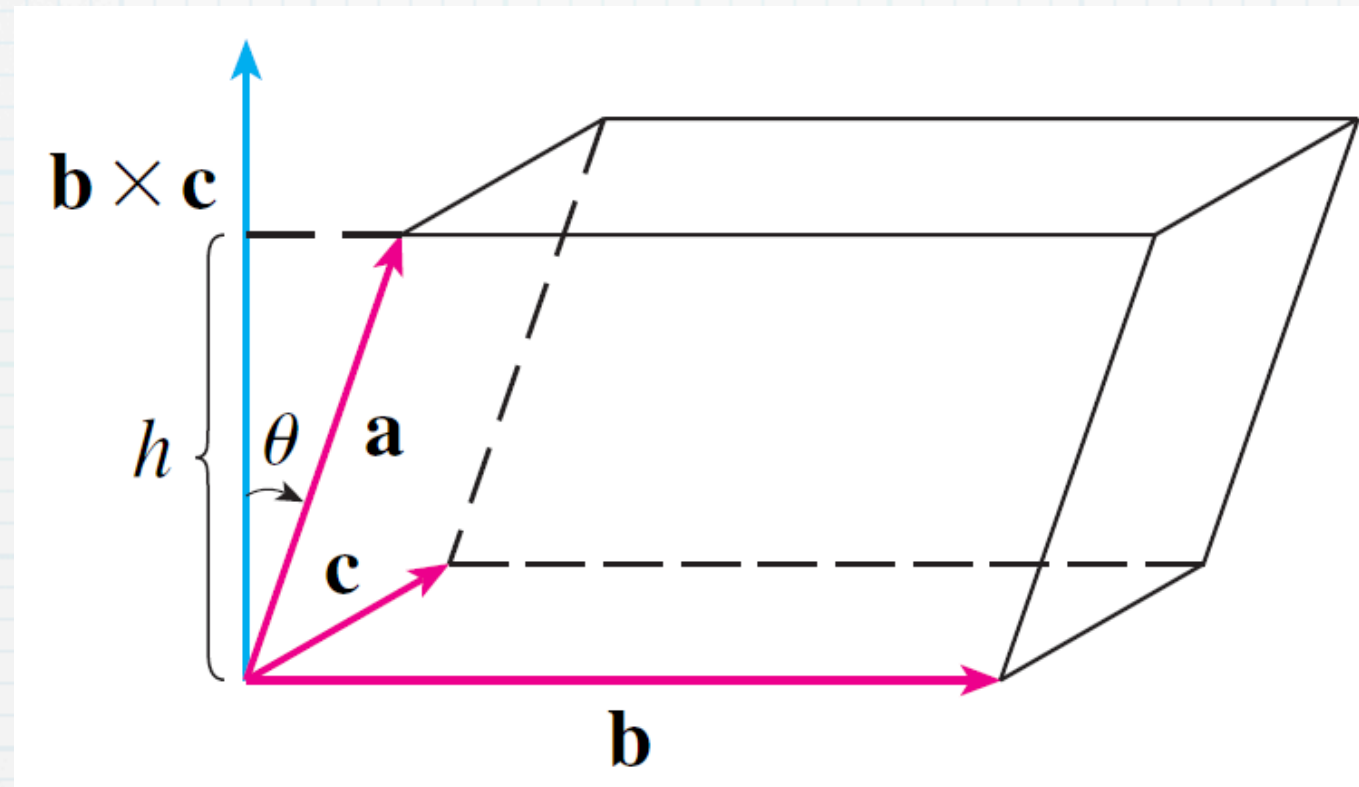
If $\mathbf{a} = \langle 1, 3, 4 \rangle$ and $\mathbf{b} = \langle 2, 7, -5 \rangle$ then

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 4 \\ 7 & -5 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & 4 \\ 2 & -5 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} \mathbf{k}$$

$$= (-15 - 28) \mathbf{i} - (-5 - 8) \mathbf{j} + (7 - 6) \mathbf{k} = -43\mathbf{i} + 13\mathbf{j} + \mathbf{k}$$

Scalar Triple Product



The volume of the parallelepiped determined by the vectors \mathbf{a} , \mathbf{b} , and \mathbf{c} is the magnitude of their scalar triple product:

$$V = | \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) |$$

Scalar Triple Product

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{a} \cdot \left[\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} \mathbf{i} - \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} \mathbf{j} + \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \mathbf{k} \right]$$

$$= a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Vector Triple Product

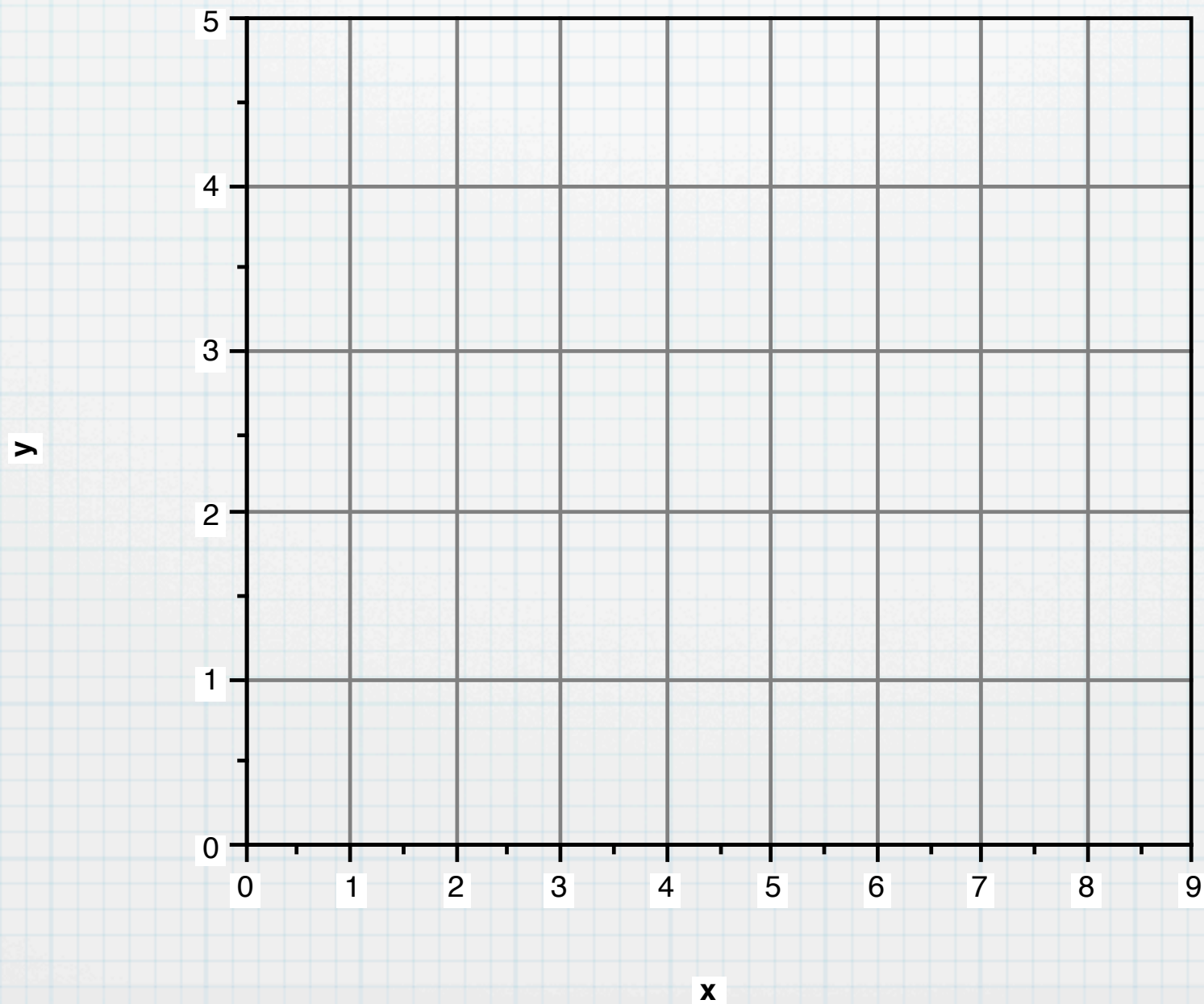
$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$$

Handouts

- * **Mathematica Notebook**
- * **Mathematica Dot and Cross Product Notebooks**
- * **Dot and Cross Product**
- * **Messing with the Cross Product**

Preview: Lines

$$\mathbf{r}(t) = \langle 4, 3 \rangle + t \langle 2, -1 \rangle$$

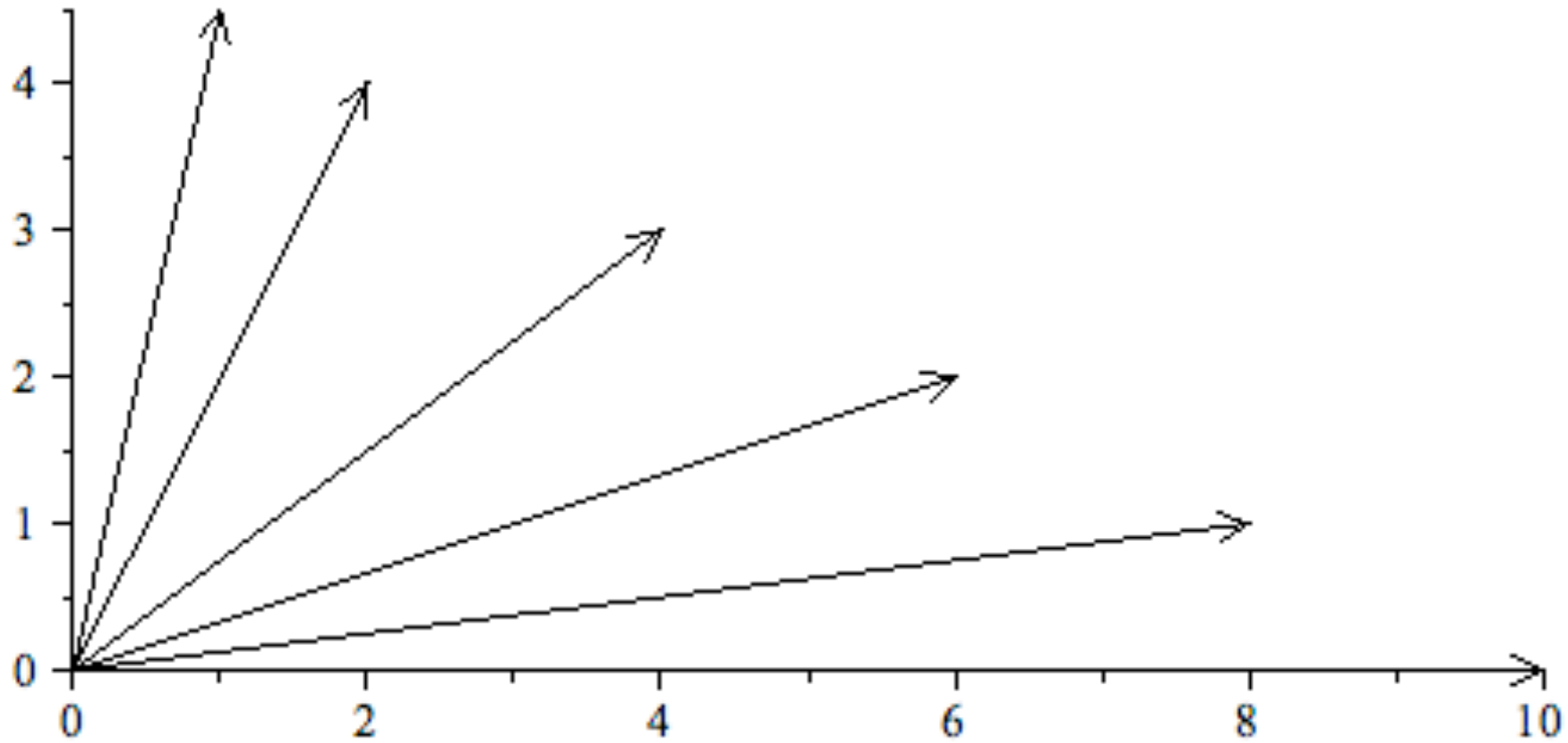


Handout

* Lines in the Plane

Lines

$$\mathbf{r}(t) = \langle 4, 3 \rangle + t \langle 2, -1 \rangle$$



Homework

- * WebAssign Assignment 2
- * Mathematica Assignment 2